Using global buckling to open an earphone carry pouch

Omar Farooq

As defined by Calladine (1989) buckling is ‘a wide range of phenomena in which structures under load cease to act in the primary fashion intended by their designers, but undergo instead an overall change in configuration’. Buckling instability can occur in members subject to compressive loads e.g. columns as a part of a building. Types of buckling include global, local and lateral torsional. In this exercise I will primarily focus on global buckling. Buckling can lead to failure of a structural element hence it is important in structural design to inhibit this limit state.

The occurrence of buckling is primarily dependant on the slenderness ratio, λ is defined as:

$$\lambda = \frac{l_e}{r_y}$$

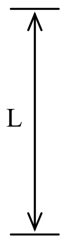
Where l_e is the effective length defined as the distance between points of inflection and r_y is the radius of gyration about the minor axis. The critical load, P_{cr} or the Euler load is the lowest value of axial load which would cause buckling.

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

Where E is the Young’s Modulus and I is the second moment of area. It can be seen that critical load is inversely proportional to the square of the effective length. Therefore as the effective length increases, the critical loading decreases. For short members the crushing load is reached before the buckling load e.g. a small piece of chalk squeezed between the fingers crushes. For slender members e.g. a long ruler, the buckling load is reached before the crushing load. From the definition of the formula it is apparent that the lowest critical load will be obtained when buckling occurs about the minor axis which has a lower second moment of area. The only material property considered within the formula is the Young’s Modulus. The term ‘EI’ can be considered as the stiffness. The stiffer the element (larger the EI) the higher the buckling load. The effective length is useful in determining the critical load for different boundary conditions. The effective length can be expressed as a factor of the length of the column:

$$L_e = kL$$

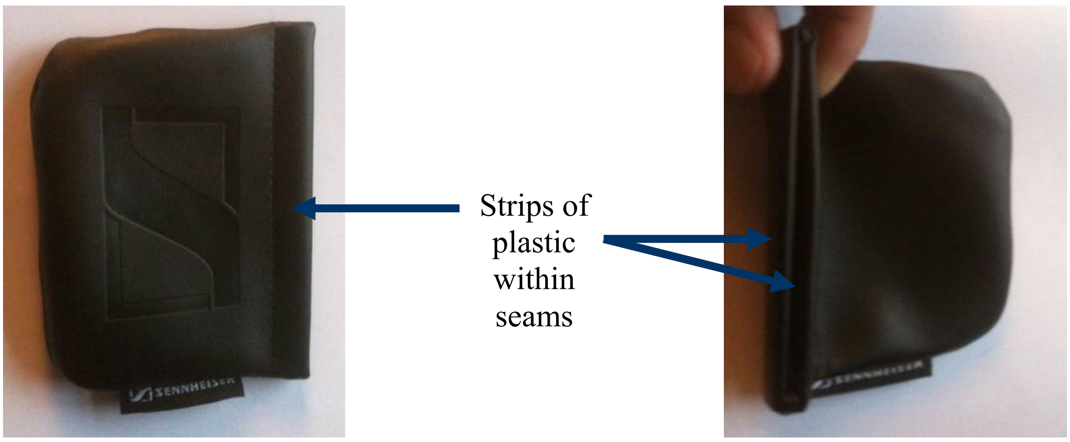
For an idealised simply supported strut with no imperfections:



$k = 1$ $Le = L$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

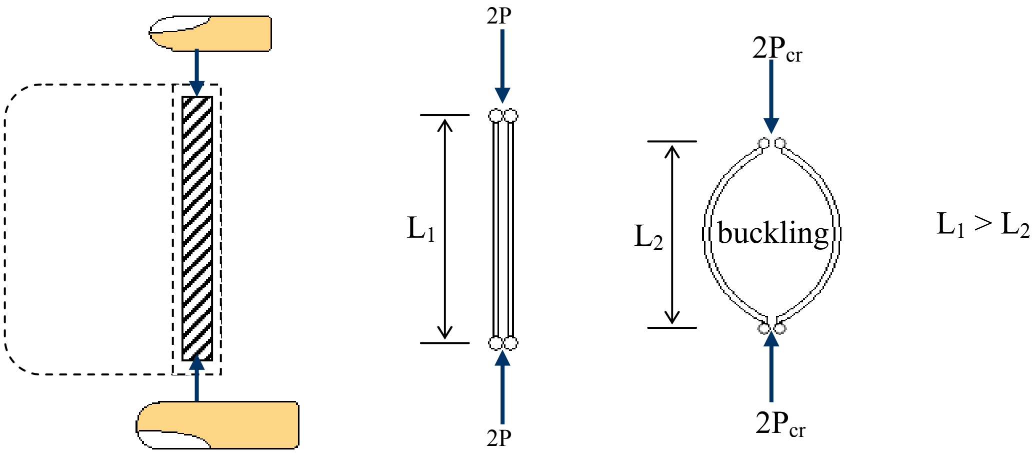
The concept of global buckling has been used innovatively as part of an earphone carry pouch. The pouch is shown in the images below. What is interesting about this example is that in this case we want the limit state to actually take place.



Stable equilibrium: When no force is applied the strips are straight and the pouch closed

Neutral equilibrium: Gradually the force applied by the fingers is increased until sudden buckling

Stable equilibrium: The strips snap back into their original configuration once the loading is removed



The sequence can be seen in more clearly in the attached videos. A simplified diagram of the operation of the pouch is shown below.

The strips are arranged so that buckling takes place about the minor axis. The fingers have two functions: firstly they act as pin supports; secondly they apply the compressive axial load to the strips. Each finger applies a force 2P with a force of P acting axially through each strip. The force is increased until the critical load P_{cr} is reached and the strips buckle. From the video it can clearly be seen that the strips buckle in a fashion similar to a pinned-column as the effective length is equal to the actual length, i.e. k=1. The strips do not behave as perfect Euler struts as they can be seen to deflect before the critical load is reached. This is suggestive of a slight initial imperfection. Once the critical loading is reached the strips can be seen to suddenly buckle. This sudden buckling causes an immediate decrease in the vertical length, causing the pouch to literally shoot-out of the fingers, as can be seen from the video. In fact the easiest method to open the pouch is by inserting a finger between the two strips. This has the effect of essentially introducing an initial imperfection. This can be analysed by considering the initial imperfection to be sinusoidal. By consideration of elasticity and equilibrium, the resulting differential equation can be solved. What is apparent from theory is that as the load is increased, deflection initially increases slowly, but then tends towards infinity as the critical load is approached. This can be clearly seen to be the case from the video. This is a much more controlled way of opening the pouch. To close the pouch one simply decreases the load applied by the fingers. The snapping of the strips is as a result of the release of elastic energy stored within the strips. The fact that the strips return to their original configuration indicates that the buckling is elastic. The above example clearly demonstrates a structural concept used innovatively.

References

Williams, M.S. Todd, J.D., 2000. *Structures theory and analysis.* 1st ed. Palgrave Macmillan.
Calladine, C. R., 1989. *Theory of shell structures.* 1st ed. Cambridge University Press