Bucking of Slender Paper Columns

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Axially loaded compression members can fail in two principal ways. When a short fat member is loaded with an increasing axial compression force it will fail through crushing or splitting of the material. This is a strength criterion. The phenomenon of buckling occurs when a long thin (slender) structural member is loaded with an increasing axial compression force. The member will deflect laterally (sideways) and fails by combined bending and compression rather than direct compression alone. This is a stiffness criterion.

The critical buckling load (P_{cr}) of a slender column is dependent on a number of member properties. Common-sense tells us that the length of the column makes a significant difference. We know that a matchstick is reasonably strong in compression (i.e we would have difficulty breaking it in our hand), but a 300m long stick of the same cross-sectional area would be very weak. In fact it is known that the buckling load of a column is proportional to inverse of the length squared. Critical buckling load is also proportional to the flexural rigidity (EI) of the cross-section. Paper columns have been used here to demonstrate the effect the length of a column has on its critical buckling load.

The columns used were cylindrical, had a total length of 156mm and a diameter of 10mm. The thickness of the wall was equal to that of a sheet of paper. Increasing load was achieved by adding water to glass atop of the columns. Figure 1 shows the initial arrangement;

Figure 1. Initial Arrangement

When the glass contained approximately 150ml of water one of the columns buckled. The failed column is shown in Figure 2, the lateral movement can be seen clearly:

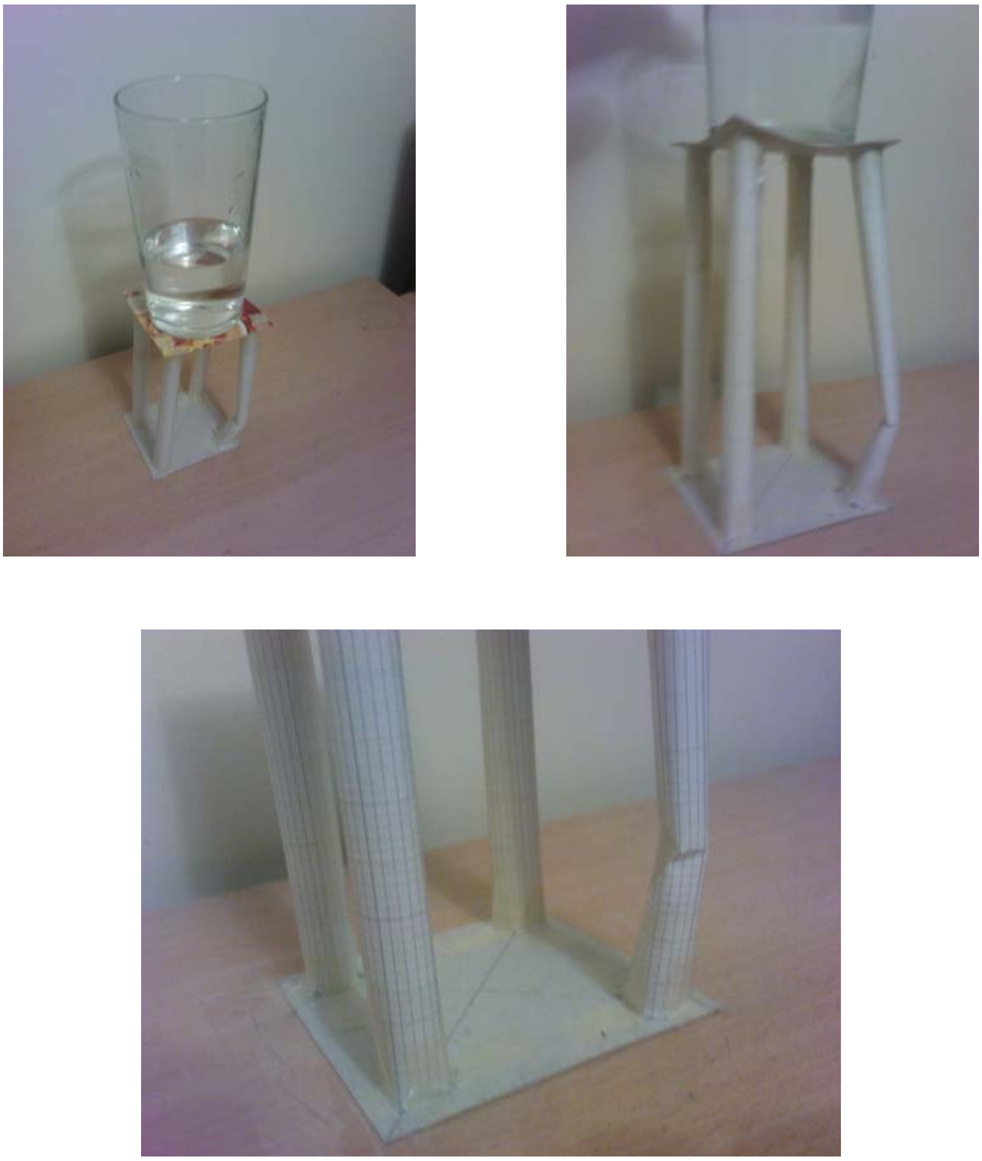

Figure 2. Failed column

An identical arrangement was then loaded to failure. However an additional strip of cardboard was introduced around all of the columns at mid height. The arrangement is shown in Figure 3:

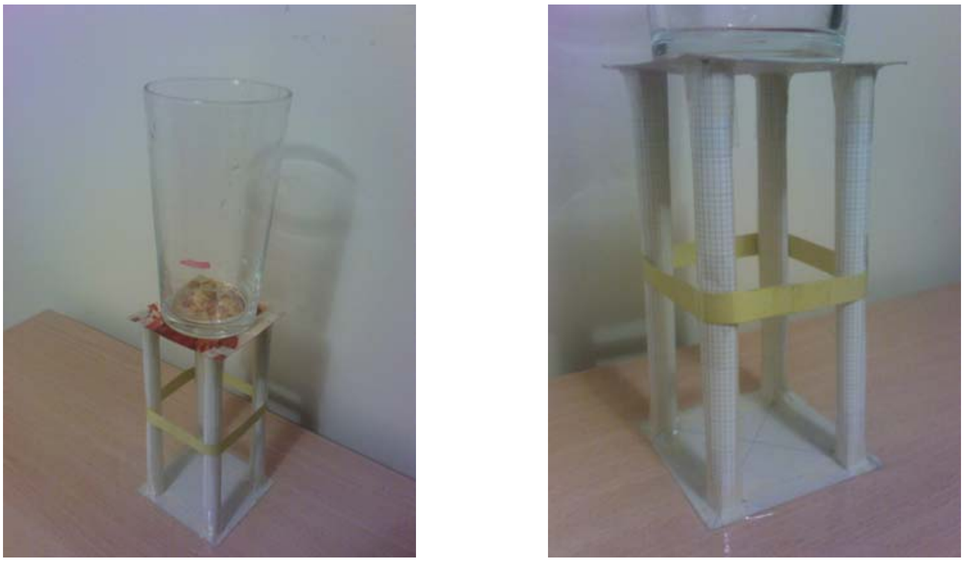

Figure 3. Improved Arrangement

The cardboard strip provided lateral restraint at mid height and essentially halved the length of the columns from 156mm to 78mm. Thus, the new arrangement was much stiffer. Figure 4 shows the arrangement supporting almost a full pint of water:

Figure 4. Improved arrangement supporting a pint of water

Some additional water was added to fill the glass causing one of the supporting columns to suddenly buckle. The failed column is shown in Figure 5:

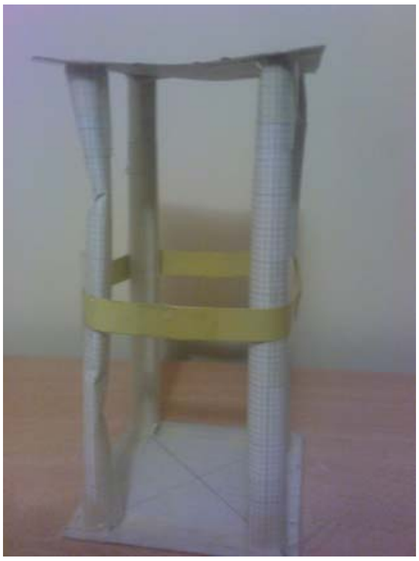

Figure 5. Failed column on improved arrangement

It can be seen that the improved arrangement can support approximately four times as much load as the initial arrangement. This is because the effective length of the columns has been halved:

$$\mathbf{1 / L^2 = 1 / 0.5^2 = 4}$$

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References

www.structuralconcepts.org

Understanding Structures, Derek Seward, Third Edition, 2003, Palgrave Macmillan